Data Matrix Disattenuation: A Simple, Effective Method for Correcting Measurement Error in Multivariate Datasets

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Abstract

In this note, I describe *data matrix disattenuation* (DMD), a method for correcting measurement error in multivariate datasets. The method employs a vector of known or estimated reliabilities to disattenuate the correlations among a set of continuous variables, then corrects the data matrix to bring it in agreement with the disattenuated correlation matrix. In doing so, it removes error from the observed scores and can substantially improve their reliability. The effectiveness of the correction increases with the number of variables in the dataset, the strength of their mutual correlations, and the size of the sample. I present initial simulation results on the performance of DMD, and provide an R function for the method. The simplicity, effectiveness, and low computational cost of DMD can make it a convenient error correction tool in many common research settings.

In real-world datasets, variables are almost always measured imperfectly, and end up containing smaller or larger amounts of error. Measurement error adds noise to individual data points, distorts the patterns of associations among variables, adds bias to parameter estimates, and reduces the power of statistical tests. While some effects of measurement error overlap with those of sampling error, they are not the same thing; increasing sample size reduces the latter but not the former, and measurement artifacts can significantly impact the results of statistical analyses even in very large datasets. Error correction is (or should be) a core task of data analysis, with unique challenges and opportunities in different disciplines.

There are many ways to deal with measurement error in multivariate datasets, including multiple variants of structural equation modeling (SEM; see Kline, 2016; Savalei, 2019). Generally speaking, measurement error can be corrected by explicitly including it into the statistical model used for the analysis; by employing specialized techniques (e.g., simulations) to first estimate and then cancel out its effect on parameter values; or by replacing the observed scores of error-contaminated variables with corrected estimates that are closer to the corresponding true scores (see Carroll et al., 2006).

In this brief note, I describe an error correction method of the latter kind. This method uses information about the amount of error in each variable to generate a corrected dataset, which is closer to the "true" dataset that would obtain under perfect measurement. I label it *data matrix disattenuation* (DMD) because it relies on the age-old technique of correction for attenuation (Spearman, 1904; see below), but applies it to an entire data matrix of continuous variables and not just to their correlation matrix. DMD is simple, straightforward, and computationally inexpensive; but it performs surprisingly well, and its simplicity makes it applicable in many common research contexts. I write this note to offer the method for inspection, feedback, and refinement—without a rigorous mathematical analysis or in-depth comparisons with other related methods. These important details remain to be worked out in the future.

Reliability and Disattenuation

I start with some basic preliminaries. In classical test theory (CTT), the observed score on a variable X can be partitioned into an unobserved *true score* component T (the measured signal) and an error component E (the noise). The *reliability* of X is defined as the proportion of the observed variance accounted for by the true score variance—or, equivalently, the squared correlation between the true and observed scores:

$$r_{XX} = \frac{V_T}{V_X} = \frac{V_T}{V_T+V_E}\,. \qquad (1)$$

(For more on reliability and the various ways in which it can be operationalized and estimated, see Mair, 2018; Meyer, 2010; Revelle & Condon, 2018.)

Information about reliability can be used to obtain corrected estimates of the observed correlations between variables, which in presence of measurement error shrink predictably

toward zero (*attenuation*). To *disattenuate* the observed correlation r_{obs} between two variables *X* and *Y*, it is sufficient to divide it by the product of the square rots of the reliabilities of *X* and *Y:*

$$r_c = \frac{r_{obs}}{\sqrt{r_{XX}}\sqrt{r_{YY}}}, \quad (2)$$

where r_c is the corrected correlation (i.e., the latent correlation between the true scores of *X* and *Y*). As a rule, both observed correlations and reliabilities are estimated empirically (for example from test-retest data, or from indices of internal consistency); the disattenuated r_c is also an estimate, whose accuracy depends on the accuracy of the quantities used to compute it (see Charles, 2005; Schmidt & Hunter, 2014; Wiernik & Dahlke, 2020; Zhang, 2022).

The Basic DMD Procedure

In a nutshell, the method I propose corrects the data matrix by bringing it in agreement with the disattenuated correlation matrix. Let $\mathbf{X}_{obs}$ be a data matrix of observed continuous variables. The DMD procedure consists of the following steps:

1. Store the means and variances of the observed variables, then standardize $\mathbf{X}_{obs}$ to yield $\mathbf{Z}_{obs}$. Calculate the observed correlation matrix $\mathbf{R}_{obs}$.

2. Use known or estimated reliabilities to obtain the disattenuated correlation matrix $\mathbf{R}_c$ (see Eq. 2). If $\mathbf{R}_c$ is non-positive definite, smooth it to a positive definite matrix (which can be done with numerical methods, e.g., Higham, 2002).

3. Obtain a *whitening matrix* $\mathbf{W}_{obs}$ from the observed correlation matrix $\mathbf{R}_{obs}$, and use it to decorrelate the variables in the standardized observed data matrix ($\mathbf{Z}_{obs}$). The ZCA method is preferred because it maximizes the correlations between the original and whitened variables (Kessy et al., 2018).

4. Obtain a *coloring matrix* $\mathbf{W}_c^{-1}$ from the disattenauted correlation matrix $\mathbf{R}_c$ (a coloring matrix is the inverse of a whitening matrix), and use it to "transfer" the disattenuated correlations onto the whitened data matrix. This yields the standardized corrected data matrix $\mathbf{Z}_c$.

5. If desired, rescale the variables in $\mathbf{Z}_c$ to the original means, with variances adjusted to reflect the correction (more on this below), yielding the unstandardized corrected data matrix $\mathbf{X}_c$.

When variables are standardized, the ZCA whitening matrix is $\mathbf{W} = \mathbf{R}^{-1/2}$; thus, steps 3 and 4 of the procedure reduce to:

$$\mathbf{Z}_c = \left(\mathbf{W}_c^{-1}\mathbf{W}_{obs}\mathbf{Z}_{obs}^{T}\right)^{T} = \left(\mathbf{R}_c^{1/2}\mathbf{R}_{obs}^{-1/2}\mathbf{Z}_{obs}^{T}\right)^{T}. \quad (3)$$

The corrected data matrix ($\mathbf{X_c}$ or $\mathbf{Z_c}$) has the same correlations as the disattenuated correlation matrix $\mathbf{R_c}$ (which may have been smoothed to ensure positive definiteness). Crucially, the corrected scores are (on average) more reliable than the original observed scores. In other words, aligning the variables to the disattenuated correlation matrix removes some of the measurement error from the variables themselves. Eq. 3 is closely related to the *moment reconstruction* method by Freedman et al. (2004), which is the most similar error correction procedure I could find in the literature (in fact, one might regard DMD as a simplified variant of moment reconstruction).

The procedure I have described is appropriate for data matrices that consist only of continuous variables. If the dataset contains two or more groups with different means, DMD should be performed separately in each group, to avoid "double counting" group differences as correlations. The resulting *un*standardized corrected matrices can then be merged back into an overall corrected matrix for further analyses.

Properties and Use of DMD

In general, the error correction provided by DMD becomes more effective as the number of variables (k) grows larger, their mutual correlations get stronger, and sample size increases (simulation results are presented below). The number of (correlated) variables in the dataset is especially critical. To illustrate: with a realistic correlational structure, a dataset whose variables have been measured with .50 reliability can be upgraded to about .60-.70 reliability with $k = 20$, and to about .80 reliability with $k = 100$. These are very large gains, that go a long way toward recovering the true scores of individual data points. If the observed scores have a reliability of .70-.80, it is easy to raise their reliability to more than .90 if there are a few dozens of variables in the dataset (for simplicity, I am assuming that all the variables have the same reliability, but this is not a requirement of the method). With enough correlated variables in the dataset, one can obtain corrected reliabilities that are arbitrarily close to one (i.e., virtually error-free scores). Notably, DMD does not require huge samples or large numbers of variables to work; on average, it yields improved reliabilities even in small datasets with only a few variables (in the limit, $k = 2$). Also, the reliabilities of observed scores do not have to be estimated with high precision; a rough approximation is sufficient to get most of the gains from the error correction procedure.

Because the correlations among the variables in the corrected matrix are disattenuated, one can fit linear models to the corrected data and obtain unbiased estimates of the parameters (assuming that reliabilities have been accurately estimated). Since the method corrects the individual data points and not just the correlation matrix, the residuals are also going to correspond more closely to those of an equivalent model fitted to the true scores. Other kinds of variable- and individual-oriented analyses can similarly benefit from the correction.

The price to pay is an increase in the variance of the disattenuated correlations, mean differences, regression coefficients, etc. This kind of bias-variance trade-off is ubiquitous in error correction methods: while corrected parameter estimates are less biased (when not entirely unbiased), they also become more uncertain and variable across samples (Carroll et al., 2006, p. 60-63). In larger samples, the resulting variance inflation can make little difference to the analysis; in smaller samples, the benefits of reduced estimation bias and improved reliability

must be carefully weighed against the corresponding increase in the uncertainty of estimates. In this regard, note that the gain in the reliability of corrected scores depends on their initial reliability and on the number of variables in the dataset, not just on sample size (see below). Simulations can be extremely useful to gauge the likely costs and benefits of DMD on a case-by-case basis.

For simple correlations and standardized mean differences, standard errors can be easily corrected (to a good approximation) by multiplying the observed SE by the disattenuation factor, which is the ratio between the corrected and observed effect sizes (see Eqs. 2 and 4). For correlations, the disattenuation factor is:

$$SE_{\mathrm{c}} = SE_{\mathrm{obs}} \frac{1}{\sqrt{r_{XX}}\sqrt{r_{YY}}}. \quad (6)$$

For standardized mean differences:

$$SE_{\mathrm{c}} = SE_{\mathrm{obs}} \frac{1}{\sqrt{r_{XX}}}. \quad (7)$$

For example, if *X* and *Y* have .50 reliability, the disattenuation factor is for correlations is 2, meaning that both the corrected correlation and the corresponding SE are going to be twice as large as those based on observed scores (Schmidt & Hunter, 2014; Wiernik & Dahlke, 2020). For more complex analyses, one can easily bootstrap the original data and apply the DMD procedure to each bootstrapped sample before computing the desired statistics; in this way, the bootstrap distribution will capture *both* sampling error and the uncertainty associated with error correction.

A simple R function (*dmd*) that performs DMD on a data matrix of continuous variables and returns the corresponding corrected matrix is available at https://doi.org/10.6084/m9.figshare.22236925.

Simulation Results

Tables 1-3 present the results of initial simulations of the method, based on normally distributed true scores and error components. The R code used for the simulations is also available at https://doi.org/10.6084/m9.figshare.22236925. Population correlation matrices were generated with the vine method (Lewandowski et al., 2009); the average absolute size of the correlations was controlled by the beta parameter. Each cell shows the average reliability of the corrected scores across 100 samples, for *N* (sample size) ranging from 100 to 5,000 and *k* (number of variables) ranging from 5 to 100.

Table 1 shows simulation results assuming that the true reliabilities in the population are known (in the simulations, all the variables in the dataset have the same reliability); the mean absolute correlations between true scores were in the .20-.28 range (beta = 4). In the white cells of the table, the correction procedure improved the reliability of every single variable in each sample. Shaded cells indicate that the corrected reliability was higher than the observed reliability at least 90% of the times (lighter shading) or at least 80% of the times (darker shading).

		k				
Pop. reliability	***N***	5	10	20	50	100
ρ_{XX} = .50	100	.53	.55	.58	.62	—
	500	.54	.58	.62	.69	.74
	1,000	.55	.58	.63	.70	.76
	5,000	.55	.58	.63	.72	.78
ρ_{XX} = .70	100	.72	.74	.76	.79	—
	500	.72	.75	.78	.83	.86
	1,000	.72	.75	.78	.83	.87
	5,000	.72	.75	.79	.84	.88
ρ_{XX} = .90	100	.90	.91	.92	.93	—
	500	.90	.91	.92	.94	.95
	1,000	.90	.91	.92	.94	.96
	5,000	.90	.91	.92	.94	.96

Table 1. Illustrative simulation results for the DMD method. Disattenuation was based on the true population reliabilities (ρ_{XX}); the mean absolute true correlation between variables was .20-.28 (beta parameter = 4). Each cell is the mean reliability of corrected scores across 100 samples. In white cells, the procedure improved the reliability of every single variable in each sample. Shaded cells indicate that the corrected reliability was higher than the observed reliability at least 90% of the times (lighter shading) or at least 80% of the times (darker shading). *N* = sample size; *k* = number of variables.

Table 2 shows analogous results when the true population reliabilities were not known a priori, but assumed to be estimated with substantial error (normally distributed with *SD* = 0.10). As can be seen by comparing these results with those in Table 1, the reliability of corrected scores was only slightly lower than in the case of known reliabilities (the difference was never larger than .01). This means that, even though it is obviously desirable to estimate reliability as accurately as possible, the choice between alternative methods (see e.g., Dunn et al., 2014; Revelle & Condon, 2018; Yang & Green, 2011) is not critical for the success of error correction.

Additional simulations (not reported here) showed that systematically *under*estimating the true reliability (which leads to overcorrection for measurement error) is less harmful than *over*estimating it (which leads to undercorrection). This is relevant because Cronbach's α (the most popular index of reliability in psychology) is a lower bound estimate of the reliability.[1] Another implication is that, if the observed reliabilities vary stochastically across samples, the reliability of the corrected scores may be maximized by deliberately introducing a slight bias toward underestimation.

Table 3 shows simulation results with known reliabilities (as in Table 1), but with somewhat stronger correlations among variables (mean absolute correlation = .28-.35, beta = 2).

[1] At the same time, note that α overestimates the proportion of variance associated with the general factor of a score if the score is not unidimensional; see Cortina (1993); Crutzen & Peters (2017); Schmitt (1996); Yang & Green (2011).

		k				
Pop. reliability	***N***	5	10	20	50	100
ρ_{XX} = .50	100	.53	.55	.58	.62	—
	500	.53	.58	.61	.68	.73
	1,000	.54	.57	.62	.69	.75
	5,000	.54	.58	.63	.71	.77
ρ_{XX} = .70	100	.72	.73	.75	.78	—
	500	.72	.74	.77	.82	.85
	1,000	.72	.75	.78	.82	.86
	5,000	.72	.75	.78	.83	.87
ρ_{XX} = .90	100	.90	.91	.91	.92	—
	500	.90	.91	.92	.93	.94
	1,000	.90	.91	.92	.93	.94
	5,000	.90	.91	.92	.93	.94

Table 2. Illustrative simulation results for the DMD method. Disattenuation was based on approximate reliabilities (normally distributed, with *SD* = 0.10 around the population value); the mean absolute true correlation between variables was .20-.28 (beta parameter = 4). Each cell is the mean reliability of corrected scores across 100 samples. In white cells, the procedure improved the reliability of every single variable in each sample. Shaded cells indicate that the corrected reliability was higher than the observed reliability at least 90% of the times (lighter shading) or at least 80% of the times (darker shading). *N* = sample size; *k* = number of variables.

		k				
Pop. reliability	***N***	5	10	20	50	100
ρ_{XX} = .50	100	.56	.60	.64	.69	—
	500	.57	.62	.68	.76	.81
	1,000	.58	.63	.69	.77	.83
	5,000	.58	.63	.70	.79	.85
ρ_{XX} = .70	100	.74	.77	.80	.83	—
	500	.75	.78	.82	.87	.90
	1,000	.75	.78	.82	.88	.91
	5,000	.75	.78	.83	.88	.92
ρ_{XX} = .90	100	.91	.92	.93	.95	—
	500	.91	.92	.94	.96	.97
	1,000	.91	.92	.94	.96	.97
	5,000	.91	.92	.94	.96	.97

Table 3. Illustrative simulation results for the DMD method. Disattenuation was based on the true population reliabilities (ρ_{XX}); the mean absolute true correlation between variables was .28-.35 (beta parameter = 2). Each cell is the mean reliability of corrected scores across 100 samples. In white cells, the procedure improved the reliability of every single variable in each sample. Shaded cells indicate that the corrected reliability was higher than the observed reliability at least 90% of the times. *N* = sample size; *k* = number of variables.

Smoothing the Corrected Correlation Matrix

The more variables there are in the dataset, and the lower their reliability is estimated to be, the higher the chance that the corrected correlation matrix $\mathbf{R}_c$ will be non-positive definite (NPD). To smooth this correlation matrix and ensure positive definiteness before computing corrected scores, one can use numerical methods; the one by Higham (2002) can be recommended as it yields matrices that are typically very close to the original, and hence near-maximal error correction. Table 4 shows the performance of Higham's (2002) method in a series of simulations. In most scenarios, the original and smoothed correlation matrices are virtually identical for practical purposes. The only cases in which one can expect substantial deviations occur when the variables have systematically low reliability (which implies a more aggressive disattenuation) *and* the sample is small relative to the number of variables.

		k				
Pop. reliability	***N***	5	10	20	50	100
$\rho_{XX} = .50$	100	31% .03 (.06)	99% .03 (.10)	100% .05 (.18)	100% .07 (.27)	—
	500	4% .01 (.01)	76% .01 (.02)	100% .02 (.06)	100% .03 (.11)	100% .03 (.14)
	1,000	1% .01 (.02)	39% .00 (.02)	100% .01 (.04)	100% .02 (.07)	100% .02 (.10)
	5,000	0% —	10% .00 (.01)	100% .00 (.01)	100% .01 (.03)	100% .01 (.04)
$\rho_{XX} = .70$	100	2% .00 (.00)	78% .01 (.03)	100% .02 (.07)	100% .03 (.12)	—
	500	0% —	25% .00 (.01)	100% .00 (.02)	100% .01 (.05)	100% .01 (.06)
	1,000	0% —	13% .00 (.00)	100% .00 (.01)	100% .01 (.03)	100% .01 (.04)
	5,000	0% —	4% .00 (.00)	91% .00 (.00)	100% .00 (.01)	100% .00 (.02)
$\rho_{XX} = .90$	100	0% —	14% .00 (.01)	100% .00 (.01)	100% .01 (.03)	—
	500	0% —	3% .00 (.00)	87% .00 (.00)	100% .00 (.01)	100% .00 (.02)
	1,000	0% —	2% .00 (.00)	78% .00 (.00)	100% .00 (.01)	100% .00 (.01)
	5,000	0% —	0% —	34% .00 (.00)	100% .00 (.00)	100% .00 (.00)

Table 4. Performance of Higham's method for smoothing non-positive definite (NPD) disattenuated correlation matrices. Disattenuation was based on the true population reliabilities (ρ_{XX}); the mean absolute true correlation between variables was .20-.28 (beta parameter = 4). Each cell shows the following results across 100 samples: the percentage of disattenuated matrices that were NPD; the average root mean square deviation (RMSD) between the original and smoothed correlations; and (in parentheses) the average largest absolute deviation (LAD) between the original and smoothed correlations. N = sample size; k = number of variables.

Scaling the Variance of Corrected Variables

There are three main options for scaling the variance of the corrected variables. The first is to standardize them to a variance of 1 (as in matrix $\mathbf{Z}_c$). The second option is to scale the variance of each variable to match the *estimated true score variance* (henceforth ETSV), so that $V_{Xc} = r_{XX}V_{X\text{obs}}$. For example, a variable with an observed variance of 4 that has been measured with a reliability of .50 will have a variance of 2 after correction and ETSV scaling (since the reliability implies that half of the observed variance is true score variance). This has the key advantage that, if the dataset contains multiple groups and DMD is performed separately by group with ETSV rescaling, the mean differences between groups will be automatically disattenuated in addition to correlations, according to:

$$d_c = \frac{d_{\text{obs}}}{\sqrt{r_{XX}}}, \tag{4}$$

where Cohen's *d* is the difference between the means of two groups, standardized by the pooled standard deviation.

The third option is to scale the variance of each variable to the *variance of estimated true scores* (henceforth VETS), so that $V_{Xc} = r_{XX}^2 V_{X\text{obs}}$. This corresponds to the variance of the estimated true scores obtained with Kelley's formula:

$$x_{\text{ETS}} = \bar{x}_{obs} + r_{XX}(x_{\text{obs}} - \bar{x}_{obs}). \tag{5}$$

For example, a variable with an observed variance of 4 that has been measured with a reliability of .50 will have a variance of 1 after correction and VETS scaling. The advantage of VETS is that corrected scores tend to yield somewhat more accurate estimates of individual true scores when there are relatively few variables in the dataset (see Table 5 below; note that ETSV equals or exceeds VETS when the number of variables is large). However, the gain in accuracy is often going to be close to negligible (see Table 5 below), and is countered by the fact that, if VETS is used to rescale the data for multiple groups, standardized mean differences will not be correctly disattenuated (as with ETSV) but inflated by a factor $\frac{1}{\sqrt{r_{XX}}}$. For this reason, I recommend using ETSV instead of VETS whenever subsequent analyses involve comparisons between groups.

Table 5 shows the average root mean squared deviation (RMSD) of observed and corrected scores from true scores, for two methods of variance scaling (ETSV and VETS). The VETS method yields somewhat more accurate scores (i.e., smaller RMSD values) when there are relatively few variables in the dataset, but begins to lose to ETSV as the number of variables becomes very large (see the column for $k = 100$). Also, the difference in accuracy between the two methods becomes negligible when the observed variables are measured with high reliability.

It is important to stress that the reliability of the corrected variables (i.e., the squared correlation with the true scores) remains always the same, regardless of the method employed to scale their variance.

				k		
Pop. reliability	**Scores**	5	10	20	50	100
ρ_{XX} = .50	Observed	1.000	1.000	1.000	1.000	1.000
	ETSV	0.728	0.690	0.643	0.569	0.505
	VETS	0.679	0.650	0.616	0.562	0.517
ρ_{XX} = .70	Observed	0.654	0.655	0.654	0.654	0.655
	ETSV	0.546	0.518	0.479	0.417	0.365
	VETS	0.526	0.501	0.468	0.416	0.372
ρ_{XX} = .90	Observed	0.333	0.333	0.333	0.333	0.333
	ETSV	0.314	0.302	0.280	0.242	0.210
	VETS	0.311	0.299	0.278	0.242	0.211

Table 5. Accuracy in the recovery of true score values for two different methods of variance rescaling, compared with observed scores. In all simulations, true scores were normally distributed and standardized (*SD* = 1). Disattenuation was based on the true population reliabilities (ρ_{XX}); the mean absolute true correlation between variables was .20-.28 (beta parameter = 4). Each cell is the average root mean square deviation (RMSD) between true scores and observed or corrected scores across 100 samples with *N* = 1,000 each. Smaller RMSD values correspond to higher accuracies.

Comparison with True Score Imputation

To better evaluate the performance of DMD, I compared it with *true score imputation* (TSI), a new method for measurement error correction that is also grounded in psychometrics (Mansolf, 2023). Like DMD, TSI requires a vector of known or estimated reliabilities for observed scores; to recover the true scores, it imputes the variables using multiple regression combined with true score estimation. The TSI method is more complex and flexible than DMD—as well as much more computationally demanding—and employs multiple imputation to explicitly incorporate uncertainty into the estimation of standard errors. To make the results of TSI comparable to those of DMD (which yields a single "best" corrected score), I calculated the reliability of mean TSI scores (obtained with the CTT model) averaged across 10 imputations (package *TSI* v. 0.1.0).

Table 6 shows some illustrative simulation results for TSI, under the same conditions of Table 1 (known population reliabilities, beta = 4). In general, mean TSI scores were strongly correlated to DMD scores (from .94 to .99). However, the TSI method struggled with small values of *N* and/or *k*; in these conditions, corrected scores were often less reliable than the original observed scores. In comparison (see Table 1), DMD tended to consistently improve reliability (however slightly) even when *N* and/or *k* were small. (Of course, this does not mean that the benefits of correction would necessarily outweigh the costs under those conditions.)

With larger sample sizes and more variables in the dataset, the reliability of mean TSI scores became very similar to that of DMD scores, with marginally lower values across the board and less consistency across variables (cell shading in Table 6). Other simulations (not reported here) showed that increasing the number of imputations beyond 10 further improved the

reliability of mean TSI scores, bringing it closer to that of DMD scores. More imputations, larger samples, and (especially) larger numbers of variables considerably increased the computation time for TSI (this is why the simulations of Table 6 are based on 10 samples per cell and stop at $k = 20$). In sum, as far as the reliability of corrected scores is concerned, DMD compared favorably to TSI, which is noteworthy in light of its comparative simplicity. Of course, TSI has other features that may recommend it over DMD, including multiple imputation and the ability to accommodate alternative measurement models (see Mansolf, 2023 for details).

			k	
Pop. reliability	***N***	5	10	20
$\rho_{XX} = .50$	100	.48	.51	.47
	1,000	.50	.55	.61
	5,000	.51	.56	.62
$\rho_{XX} = .70$	100	.69	.71	.73
	1,000	.71	.73	.77
	5,000	.70	.74	.78
$\rho_{XX} = .90$	100	.89	.91	.91
	1,000	.90	.90	.92
	5,000	.90	.91	.92

Table 6. Illustrative simulation results for the TSI method. Computations were based on the true population reliabilities (ρ_{XX}); the mean absolute true correlation between variables was .20-.28 (beta parameter = 4). Each cell is the mean reliability of corrected scores across 10 samples (scores are calculated as the mean of 10 imputations). In white cells, the procedure improved the reliability of every single variable in each sample. Shaded cells indicate that the corrected reliability was higher than the observed reliability at least 90% of the times (lighter shading), at least 80% of the times (medium shading), at least 50% of the times (darker shading), or less than 50% of the times (black). N = sample size; k = number of variables.

Conclusion

To conclude: data matrix disattenuation (DMD) is an effective method for correcting measurement error in multivariate datasets. It does not require complex analytic choices or unwieldy computations; all that users need to provide is a vector of reliabilities (which do not have to be estimated very precisely for the method to work). For some applications, it may be sufficient to apply DMD and then analyze the corrected data matrix; in other cases, there will be the need for additional computations (e.g., bootstrapped SEs and confidence intervals for model parameters). I believe DMD has the potential to become a useful addition to the toolkit of error correction methods, especially for practicing researchers in search of simple, efficient solutions that can be applied to common data-analytic scenarios.

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